A multivariate multilevel longitudinal functional model

for repeatedly observed human movement data

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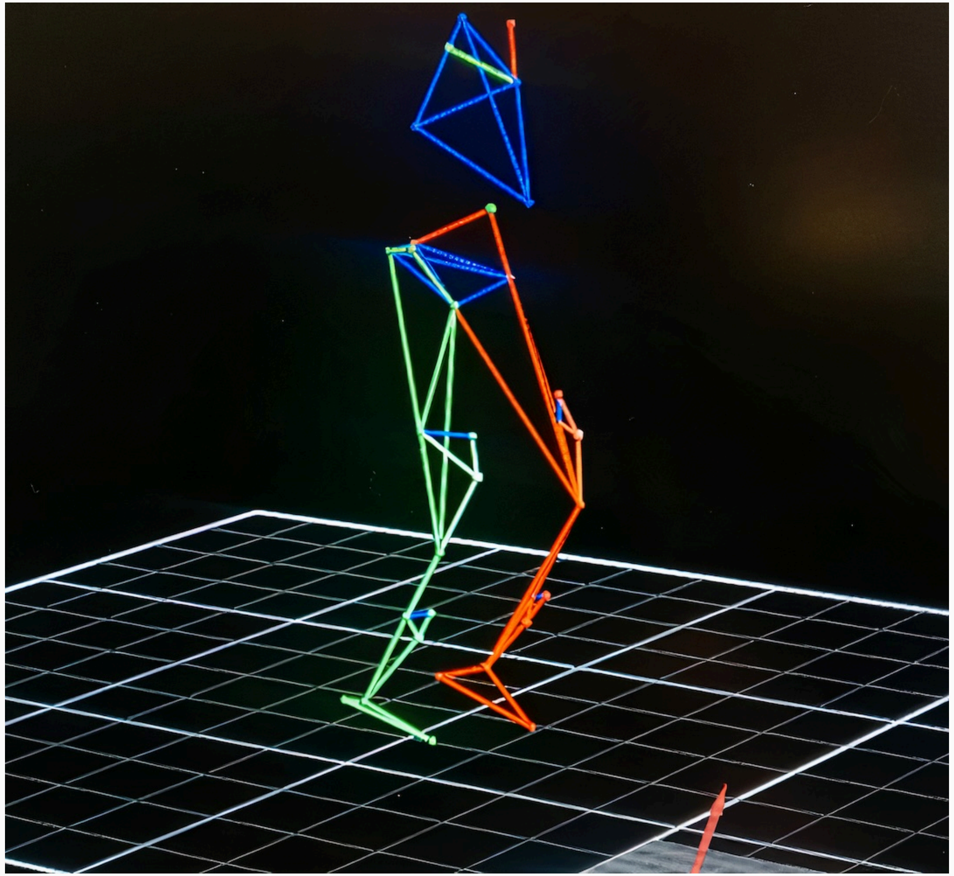
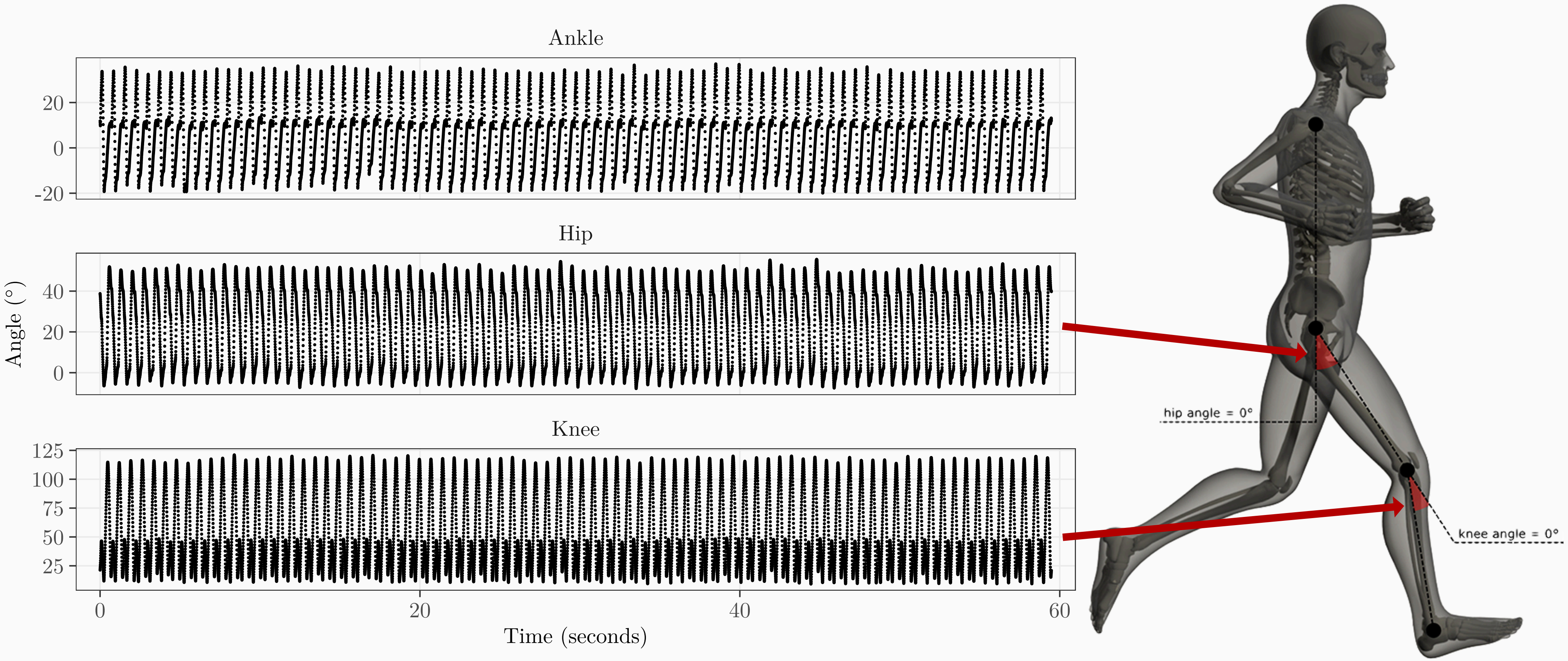
55es Journée de Statistique de la SFDS
May 28th, 2024

RISC dataset

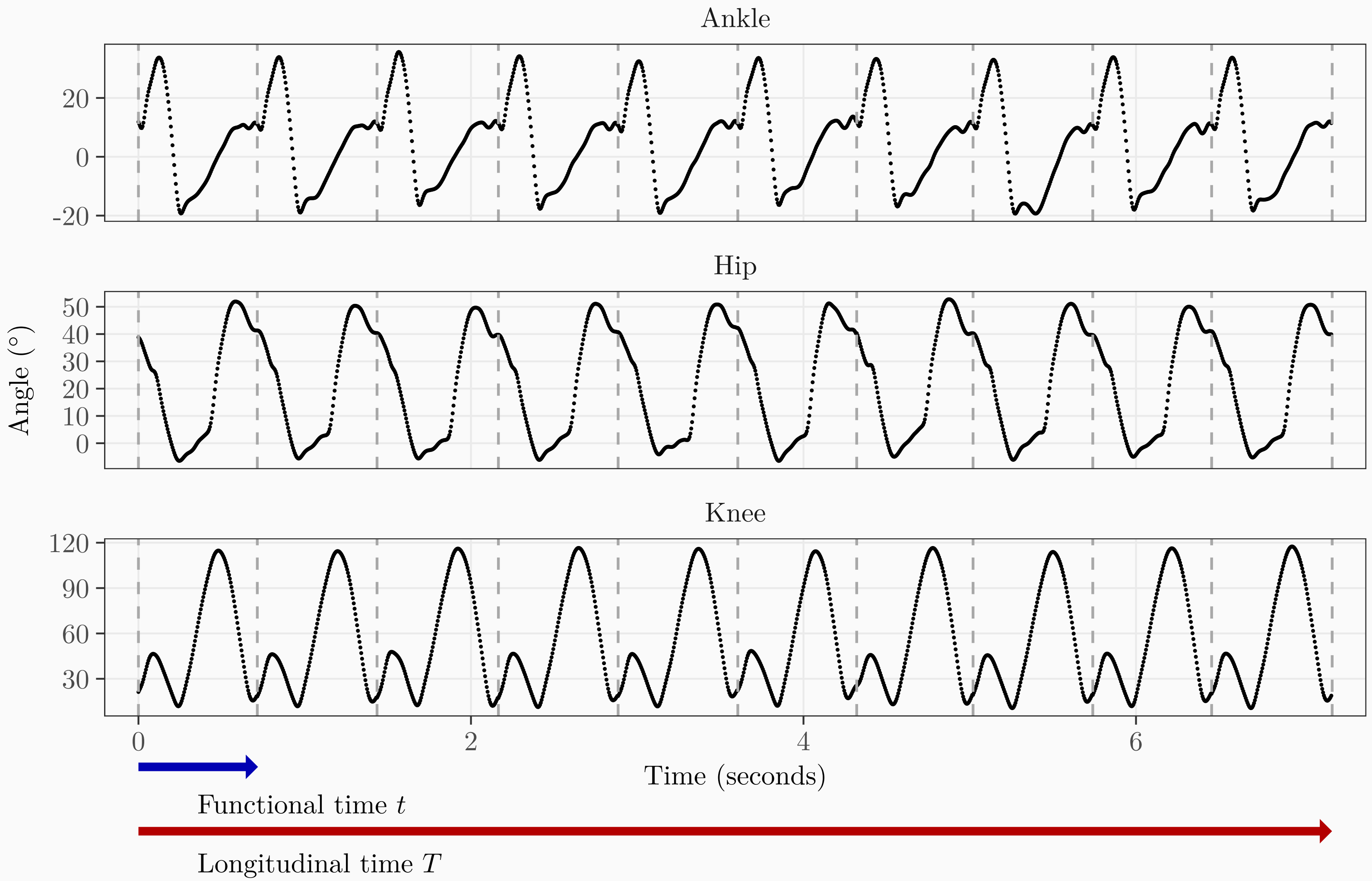



@DCU_RISC_Study

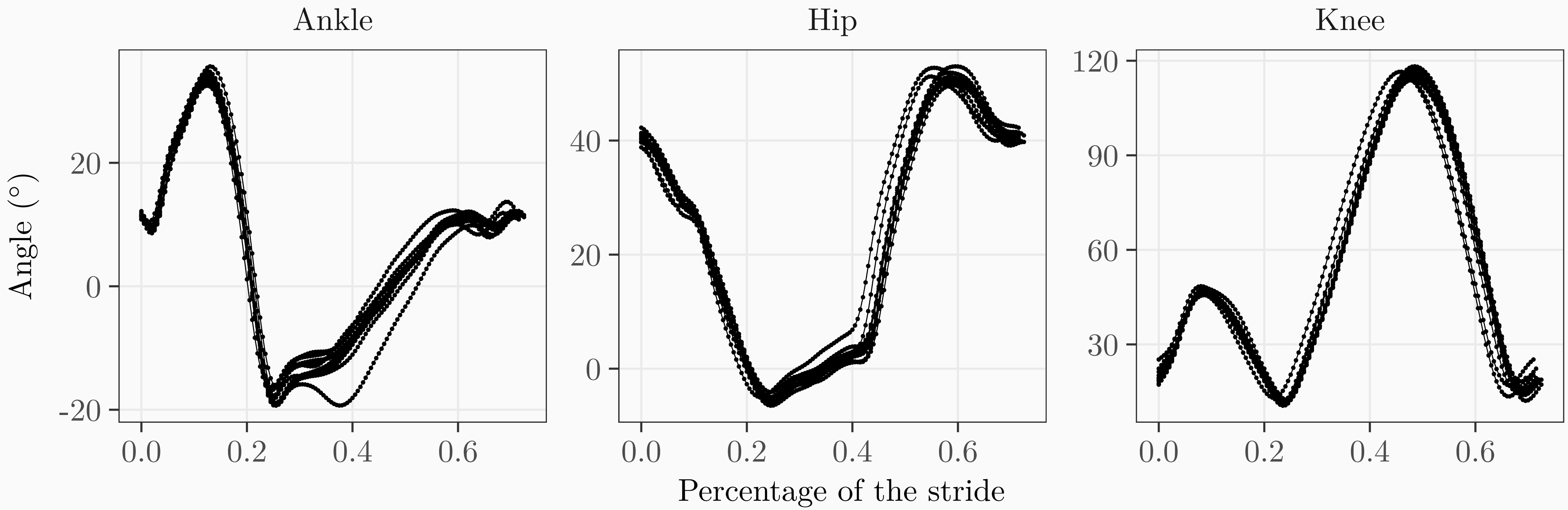
(Almost) Raw data



Segmented data

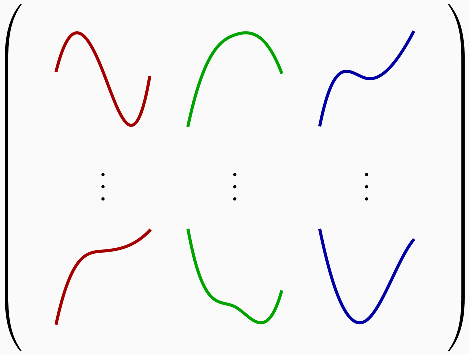


Segmented data

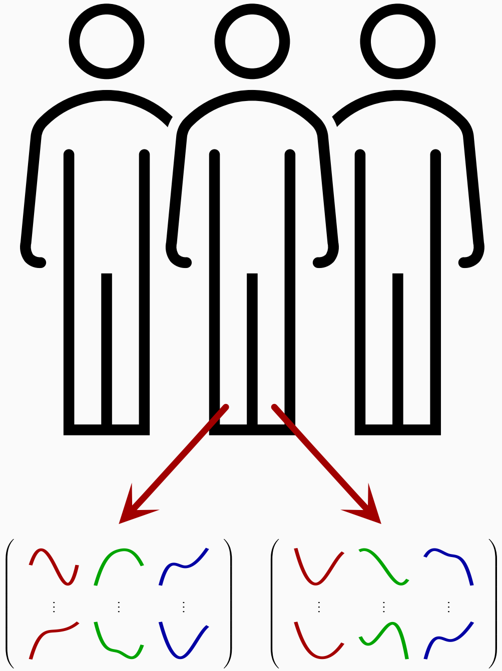


Data characteristics

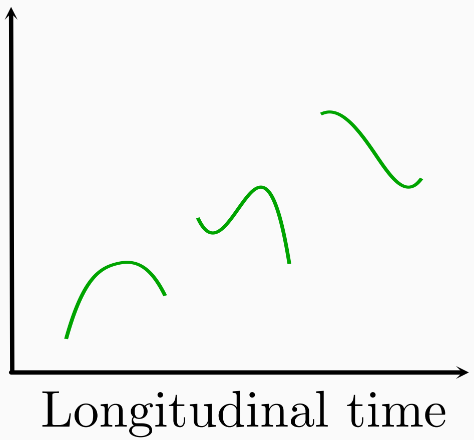
Multivariate



Multilevel



Longitudinal



Multivariate multilevel longitudinal functional model

$$\mathbf{y}_{ijl}(t) = \mu(\mathbf{x}_{ijl}, t) + \mathbf{u}_i(t, T_{ijl}) + v_{ij}(t, T_{ijl}) + \varepsilon_{ijl}(t)$$

$$\mathbf{y}_{ijl}(t) = \left(y_{ijl}^{(hip)}(t), y_{ijl}^{(knee)}(t), y_{ijl}^{(ankle)}(t)\right)^\top$$

$$i = 1, \ldots, N$$

$$j = \{\text{left}, \text{right}\}$$

$$l = 1, \ldots, n_{ij}$$

$$t \in [0, 100\%]$$

Multivariate multilevel longitudinal functional model

$$\mathbf{y}_{ijl}(t) = \mu(\mathbf{x}_{ijl}, t) + \mathbf{u}_i(t, T_{ijl}) + v_{ij}(t, T_{ijl}) + \varepsilon_{ijl}(t)$$

"Fixed effects"
Mean function and effect of scalar covariates,
e.g. speed, sex, injuries, ...

Multivariate multilevel longitudinal functional model

$$T \in [0, 1]$$

$$\mathbf{y}_{ijl}(t) = \mu(\mathbf{x}_{ijl}, t) + \mathbf{u}_i(t, T_{ijl}) + v_{ij}(t, T_{ijl}) + \varepsilon_{ijl}(t)$$

Subject and side mean

Subject mean

Multivariate multilevel longitudinal functional model

$$\mathbf{y}_{ijl}(t) = \mu(\mathbf{x}_{ijl}, t) + \mathbf{u}_i(t, T_{ijl}) + v_{ij}(t, T_{ijl}) + \varepsilon_{ijl}(t)$$

Smooth random error

Multivariate multilevel longitudinal functional model

$$\mathbf{y}_{ijl}(t) = \mu(\mathbf{x}_{ijl}, t) + \mathbf{u}_i(t, T_{ijl}) + v_{ij}(t, T_{ijl}) + \varepsilon_{ijl}(t)$$

Assume an estimate has been subtracted

Focus on modeling this part

→ Complex functional model, varying over two timescales t and T

Key ideas

$$\mathbf{y}_{ijl}(t) = \sum_{k=1}^{K} y^{\star}_{ijl,k}\, \phi_k(t)$$

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Multivariate basis functions that do not depend on longitudinal T

For univariate longitudinal functional data:

> Park and Staicu (2015)
> Lee et al. (2019)
> Li et al. (2022)

Key ideas

$$\mathbf{y}_{ijl}(t) = \sum_{k=1}^{K} y^{\star}_{ijl,k} \phi_k(t)$$

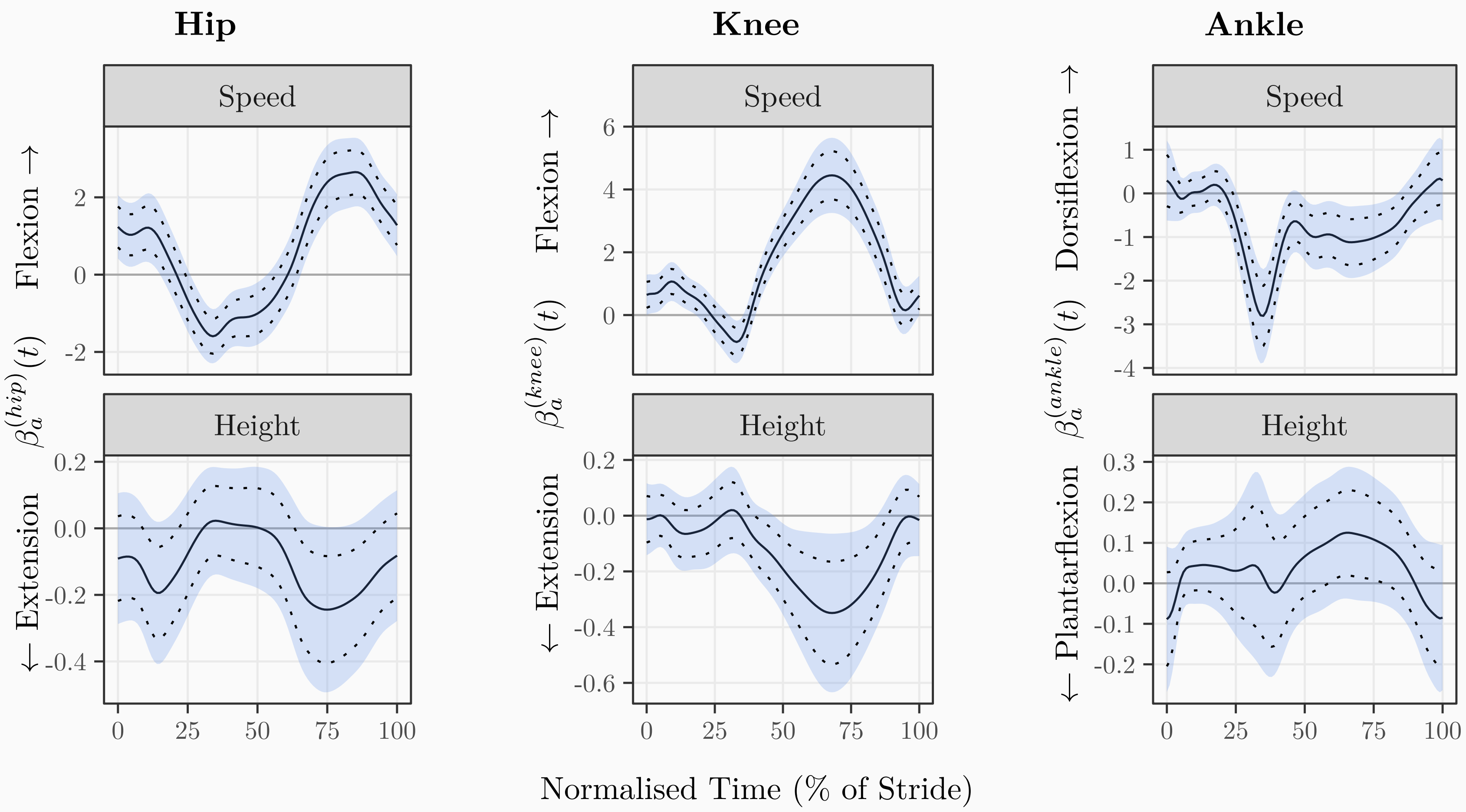
Basis coefficients capturing the longitudinal trends

$$y^{\star}_{ijl,k} = u^{\star}_{i,k}(T_{ijl}) + v^{\star}_{ij,k}(T_{ijl}) + \epsilon^{\star}_{ijl,k}$$

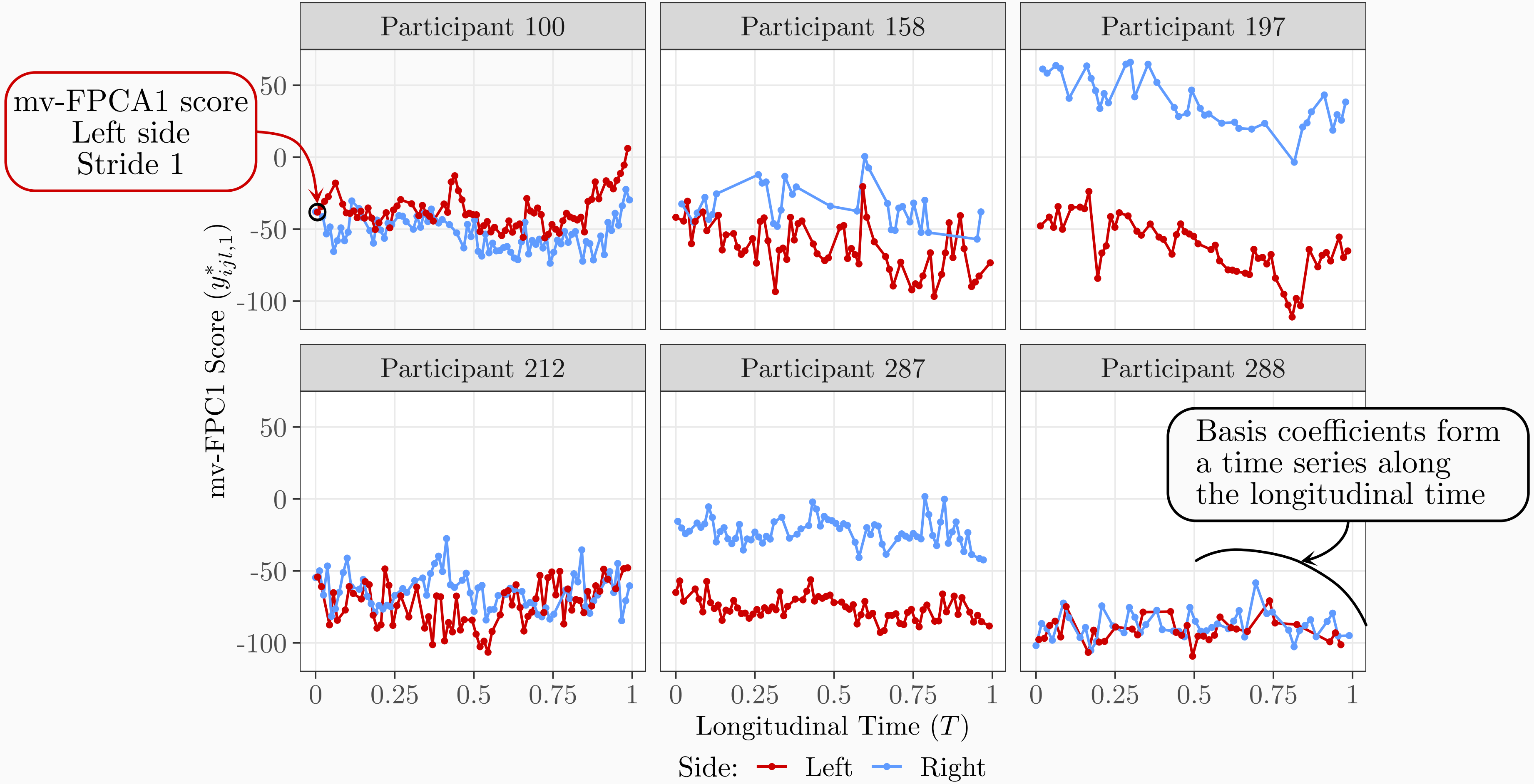
Functional Multilevel model (Di et al., 2009):

> For each of the basis coefficients k
> $T =$ longitudinal time

Fixed effects



ml-FPCA of the mv-FPCA scores



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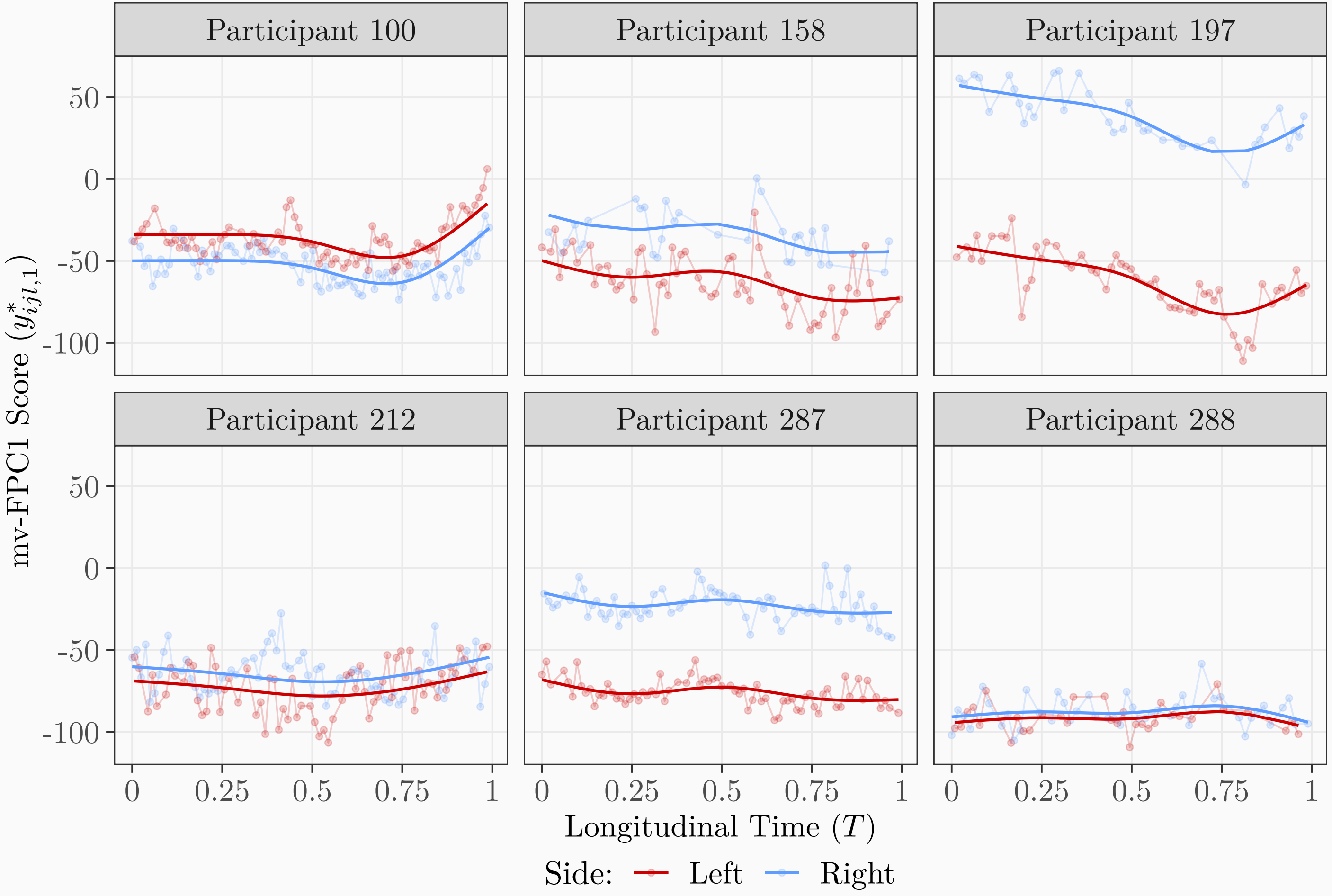
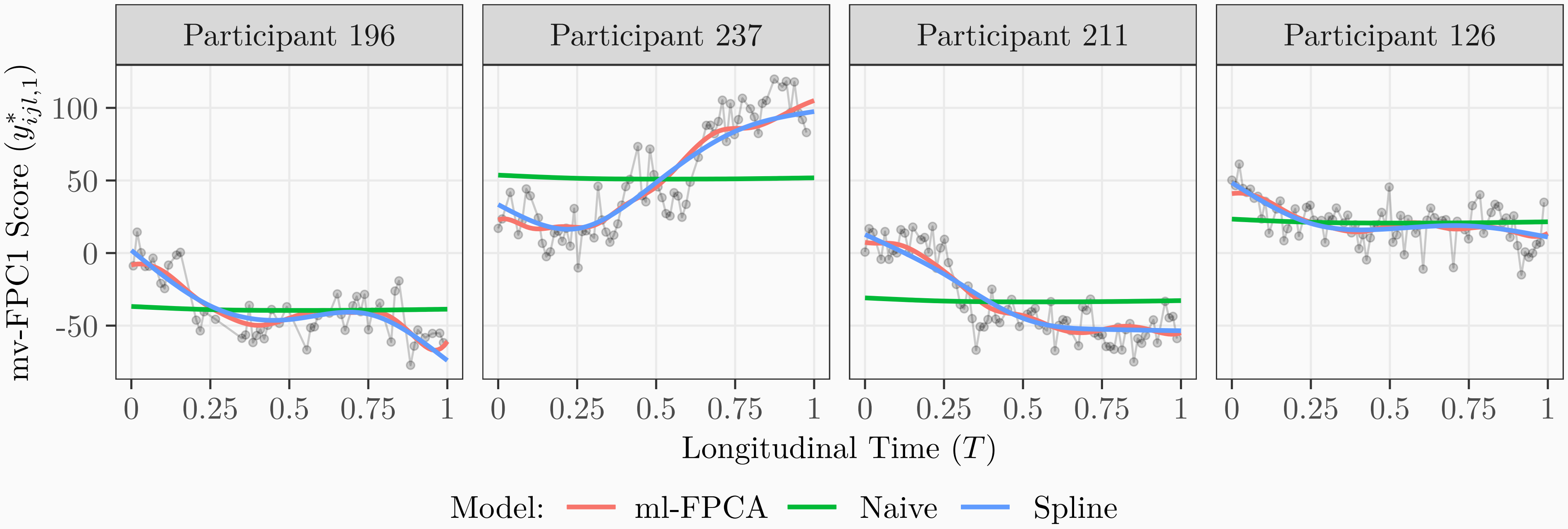


ml-FPCA of the mv-FPCA scores



Individual analysis for one participant

Takeaway ideas

- We decomposed the main sources of variability in the data by:
  - capturing multivariate functional dependenceusing a pooled mv-FPCA basis;
  - modeling the multilevel longitudinal trends through the mv-FPCA scores.
- We identified very simple longitudinal trends.
- It can be improved by relaxing and imposing assumptions.

Thank you for your attention!

References

Di, C., C. M. Crainiceanu, B. S. Caffo, et al. (2009). "Multilevel Functional Principal Component Analysis". In: The annals of applied statistics 3.1, pp. 458–488. ISSN: 1932-6157. DOI: 10.1214/08-AOAS206SUPP.

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