Stats, Math and AI

An introduction to object data analysis

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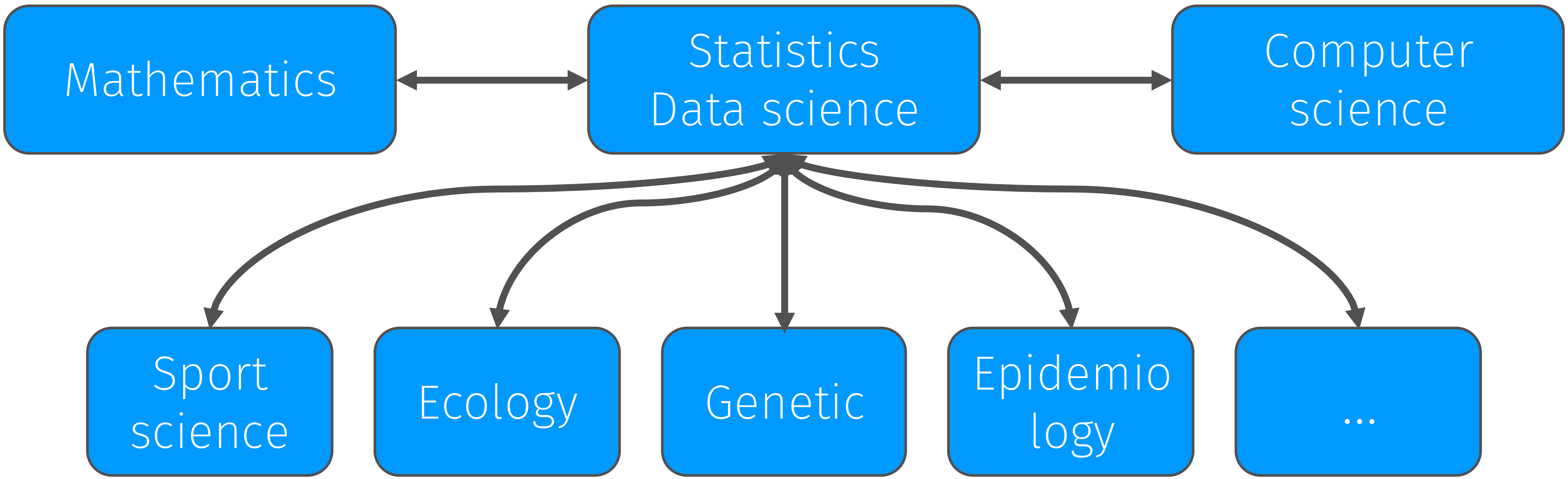


The best thing about being a statistician is that you get to play in everyone's backyard. — John Tukey[1]

[1]see www.princeton.edu/pr/news/00/q3/0727-tukey.htm or The New York Times, July 28, 2000.

AI is the science of making machines capable of performing tasks that would require intelligence if done by humans. — Marvin Minsky[1]

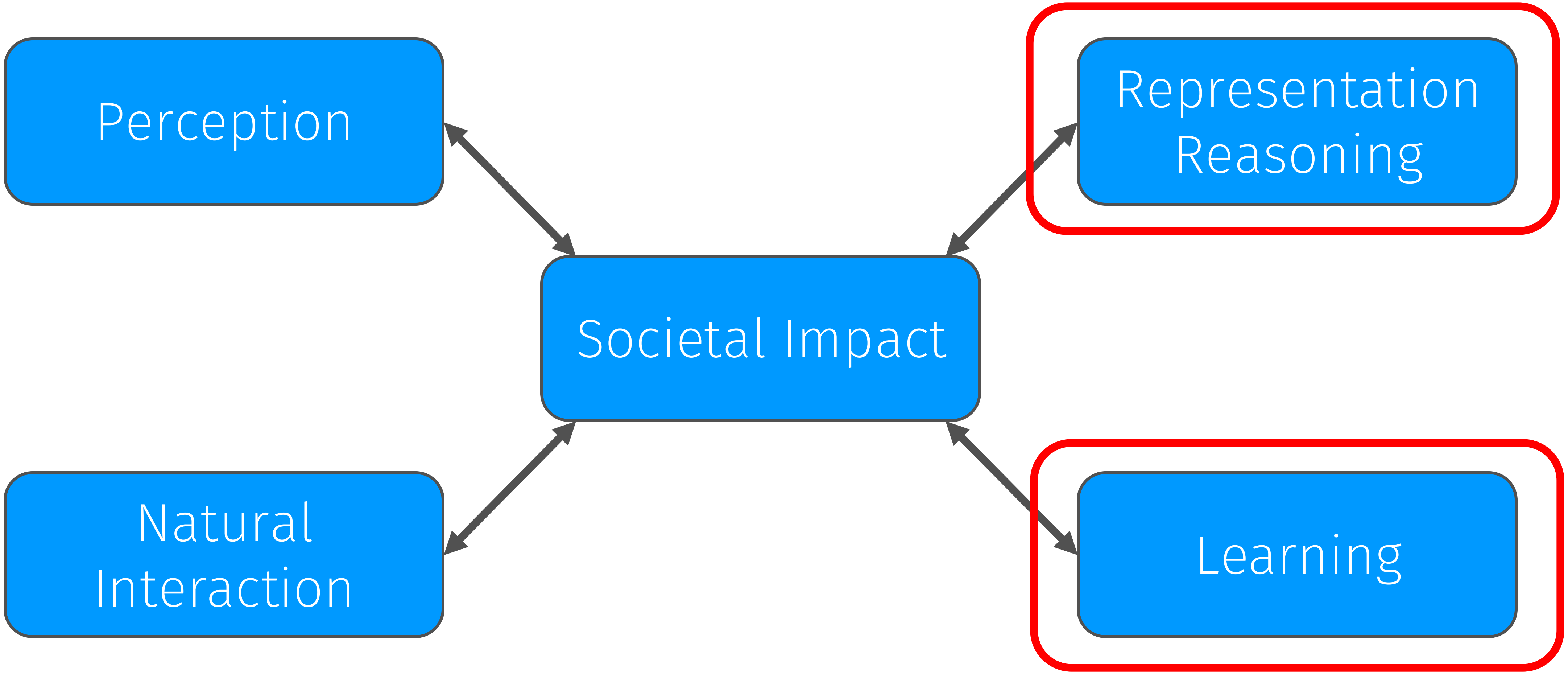
AI is the science and engineering of making machines do tasks they have never seen and have not been prepared for beforehand. — José Hernández-Orallo[2]

The human mind is not merely a collection of special-purpose programs hard-coded by evolution. The mind is not a single, general-purpose “blank slate” system capable of learning anything from experience. — François Chollet

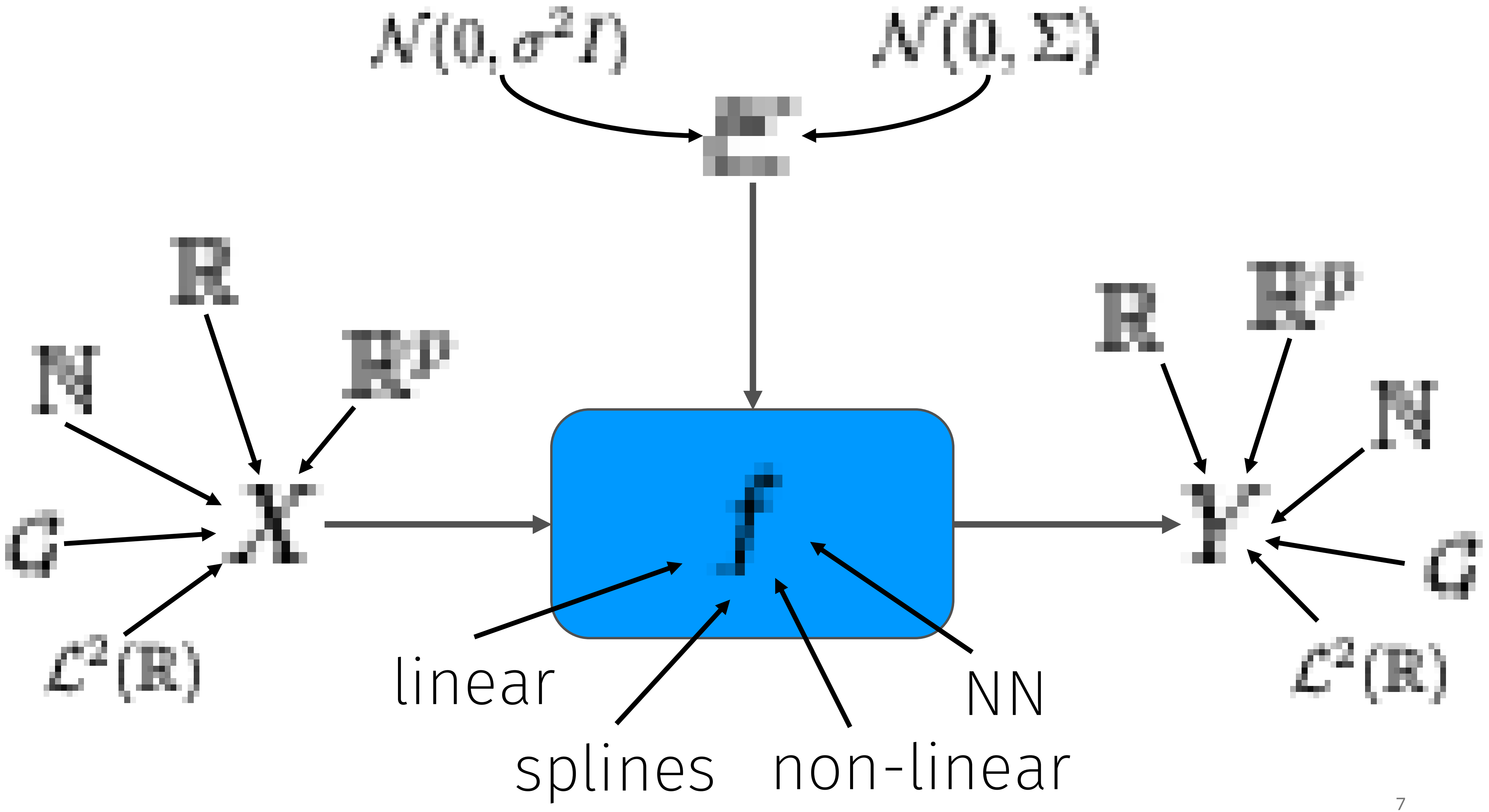
[1]Minsky, M. L. Semantic information processing. Cambridge, MIT Press (2003).

[2]Hernández-Orallo, J. Evaluation in artificial intelligence: from task-oriented to ability-oriented measurement. Artificial Intelligence Review (2017).

[3] Chollet, F. On the measure on intelligence (2019).



[1]Minsky, M. L. Semantic information processing. Cambridge, MIT Press (2003).


$\mathcal{N}(0, \sigma^2 I)$
$\mathcal{N}(0, \Sigma)$
$\mathbf{E}$
$\mathbb{R}$
$\mathbb{N}$
$\mathbb{R}^p$
$\mathbb{C}$
$\mathcal{L}^2(\mathbb{R})$
$\mathbf{X}$
f
$\mathbf{Y}$
$\mathbb{R}$
$\mathbb{R}^p$
$\mathbb{N}$
$\mathbb{C}$
$\mathcal{L}^2(\mathbb{R})$
linear
NN
splines
non-linear

$$Y = f(X) + \varepsilon$$

Loss function:

$$\mathcal{L}(Y, f(X)) = (Y - f(X))^2$$

Solution:

$$f(X) = \mathbb{E}(Y \mid X = x)$$

K-nearest neighbors

$$Y = f(X) + \varepsilon$$

$$f(X) = \frac{1}{k} \sum_{i \in N_k(X)} Y_i$$

Linear regression

$$Y = f(X) + \varepsilon$$

$$f(X) = X\beta$$

Student's test

$$Y = f(X) + \varepsilon$$

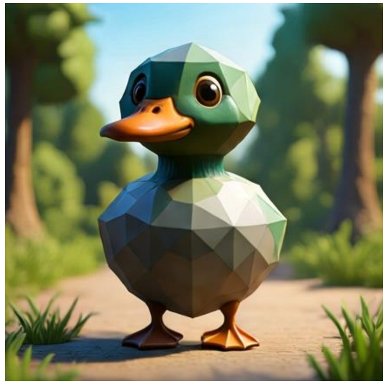
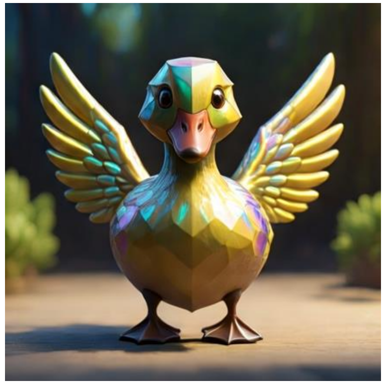
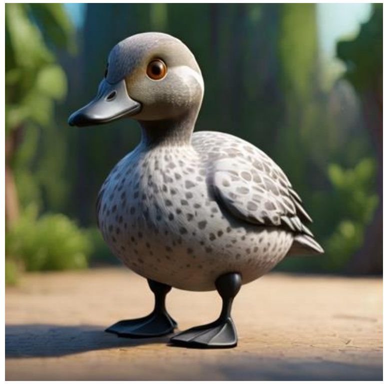
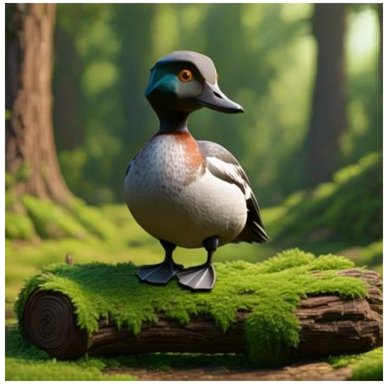
$$f(X) = \beta_0 + \beta_1 X, \quad X \in \{0, 1\}$$

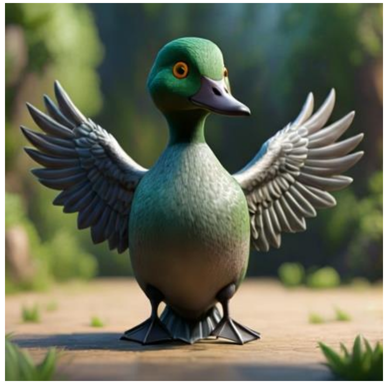
Deep neural network

$$Y = f(X) + \varepsilon$$

$$f(X) = W^{(L)}\sigma(W^{(L-1)}\sigma(\cdots\sigma(W^{(1)}X + b^{(1)})\cdots) + b^{(L-1)}) + b^{(L)}$$

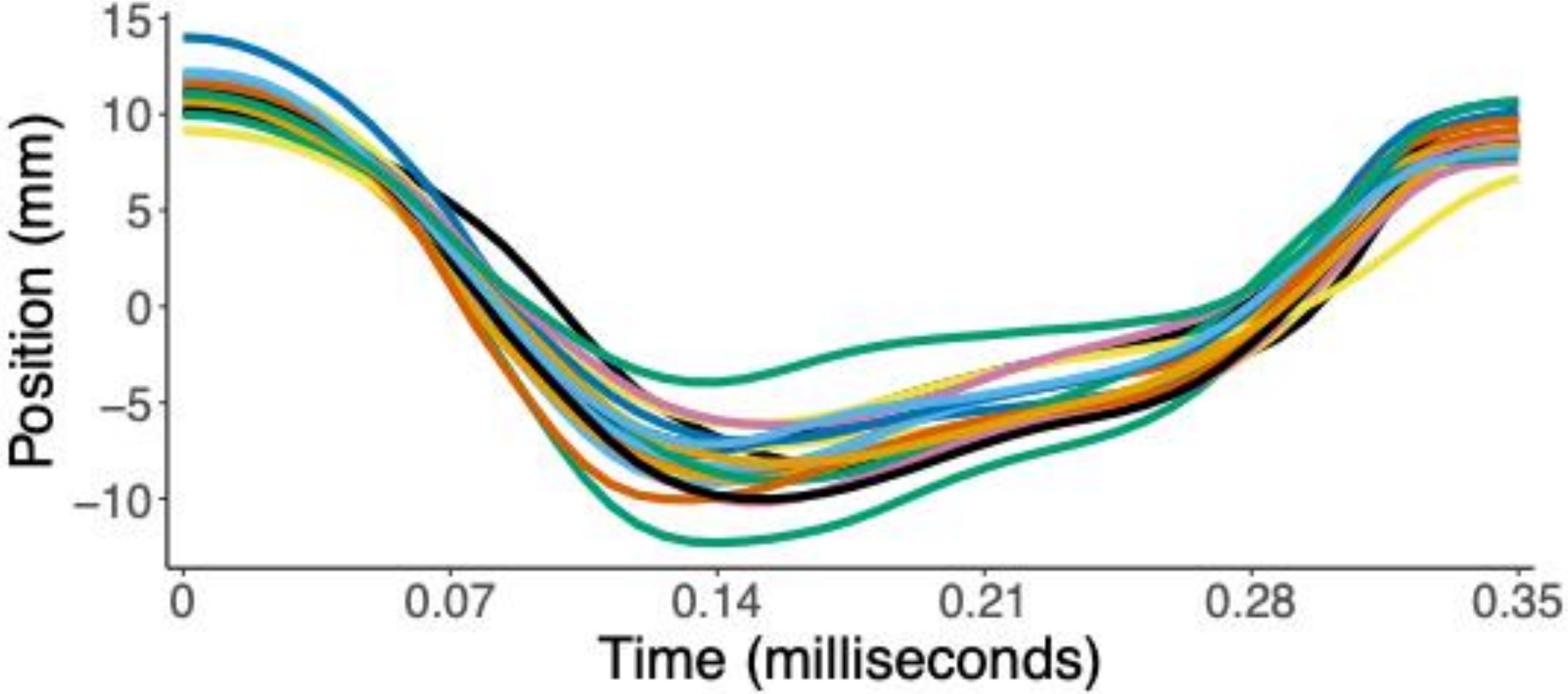
~~$\mathcal{X} = \{duck\}$~~



Ceci n'est pas un canard.

Generated using Apple Intelligence.



Ramsay, J. and Silverman B., Functional Data Analysis, Springer (2005).

Karhunen-Loève decomposition

$$X(t) = \sum_k a_k \phi_k(t)$$

Solution:

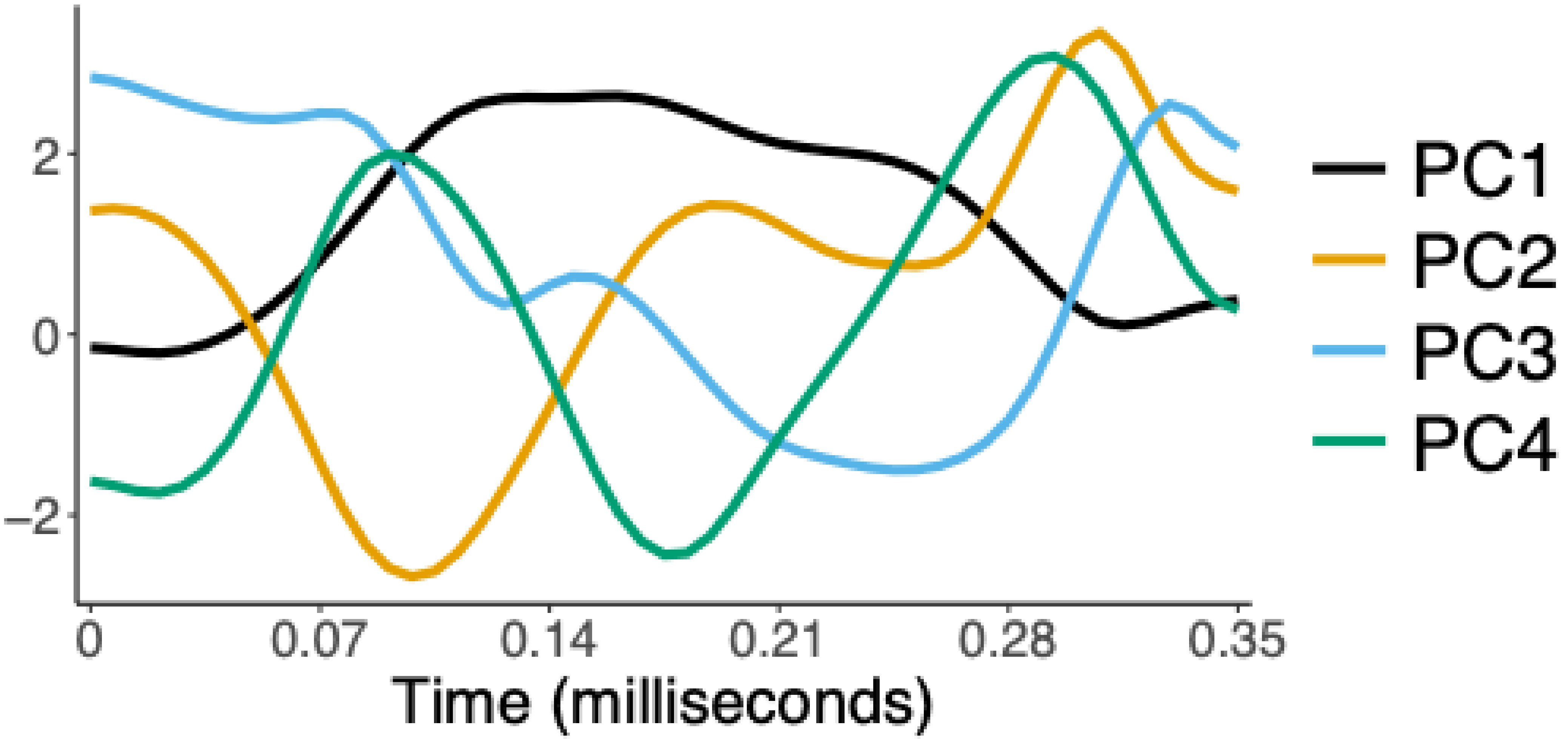
ϕ_k eigenfunctions of the covariance function C

$a_k = \langle X, \phi_k \rangle$

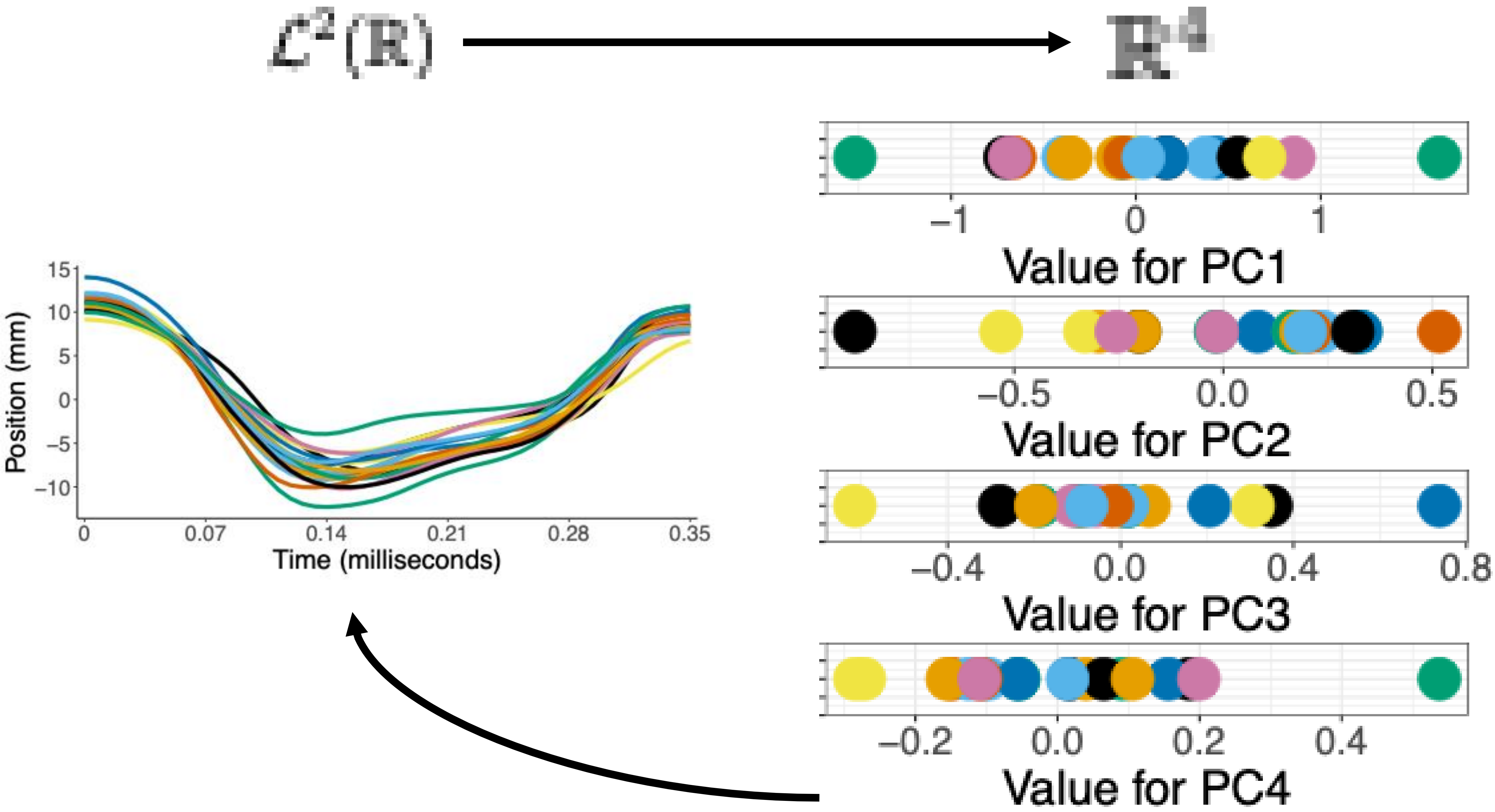
Proof:

Study of the operator $\Gamma f = \int C(s, \cdot) f(s) \mathrm{d}s$

Happ, C. and Greven, S., Multivariate Functional Principal Component Analysis for Data Observed on Different (Dimensional) Domains (2018).



2
0
−2
0
0.07
0.14
0.21
0.28
0.35
Time (milliseconds)
PC1
PC2
PC3
PC4


$\mathcal{L}^2(\mathbb{R})$
$\mathbb{R}^4$
Position (mm)
Time (milliseconds)
15
10
5
0
−5
−10
0
0.07
0.14
0.21
0.28
0.35
−1
0
1
Value for PC1
−0.5
0.0
0.5
Value for PC2
−0.4
0.0
0.4
0.8
Value for PC3
−0.2
0.0
0.2
0.4
Value for PC4

$$Y = f(X) + \varepsilon$$

$$Y = \langle X, \phi \rangle$$

$$Y = \langle X, \sum_k a_k X_k \rangle = \sum_k a_k \langle X, X_k \rangle$$

Golovkine, S. et al., On the use of the Gram matrix for multivariate functional principal components analysis (2025).

Principal differential analysis

$$L = \beta_0 I + \beta_1 D + D^2$$

$$\frac{d^2}{dt^2} X(t) = -\beta_1(t) \frac{d}{dt} X(t) - \beta_0(t) X(t)$$

Objective:

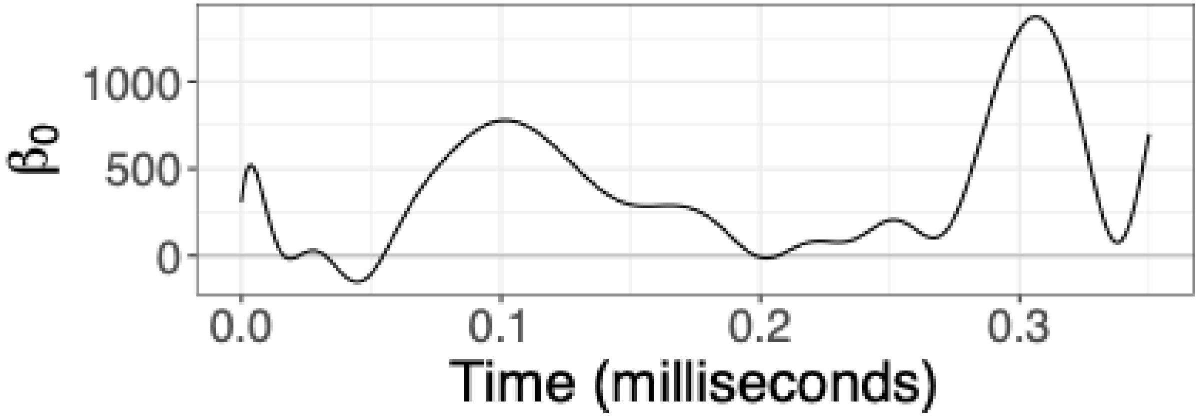
Find L such that $LX \approx 0$

Find ϕ such that $L\phi = 0$

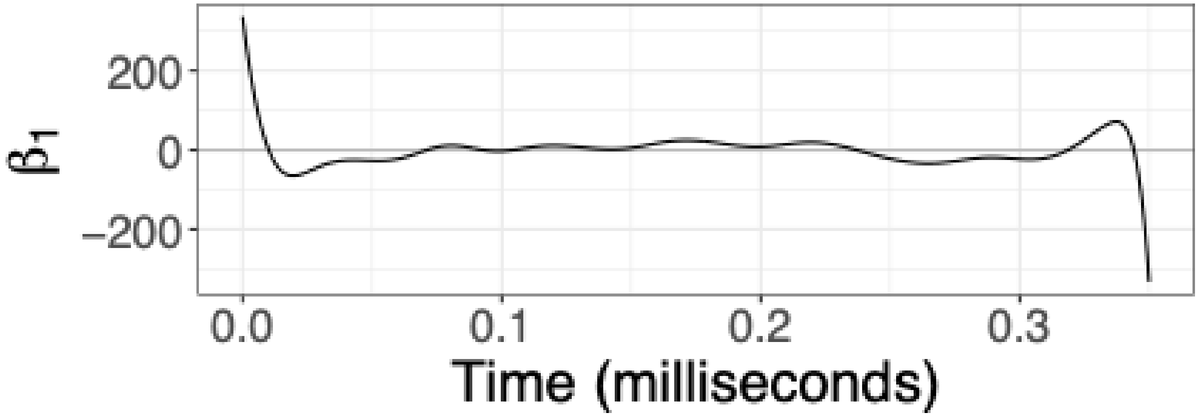
Solution:

Estimation by minimization of least-squares.

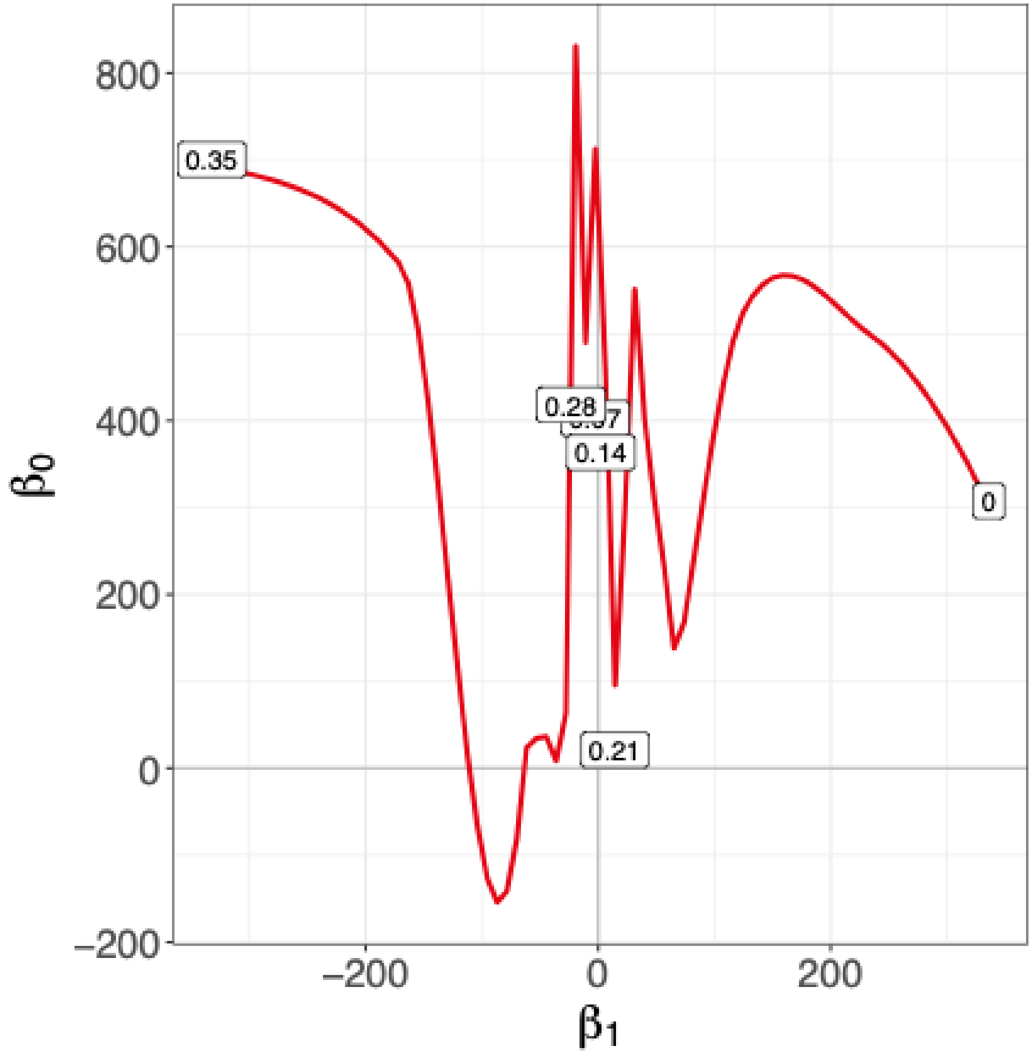
Ramsay, J. O., Principal Differential Analysis: Data Reduction by Differential Operators (1996).



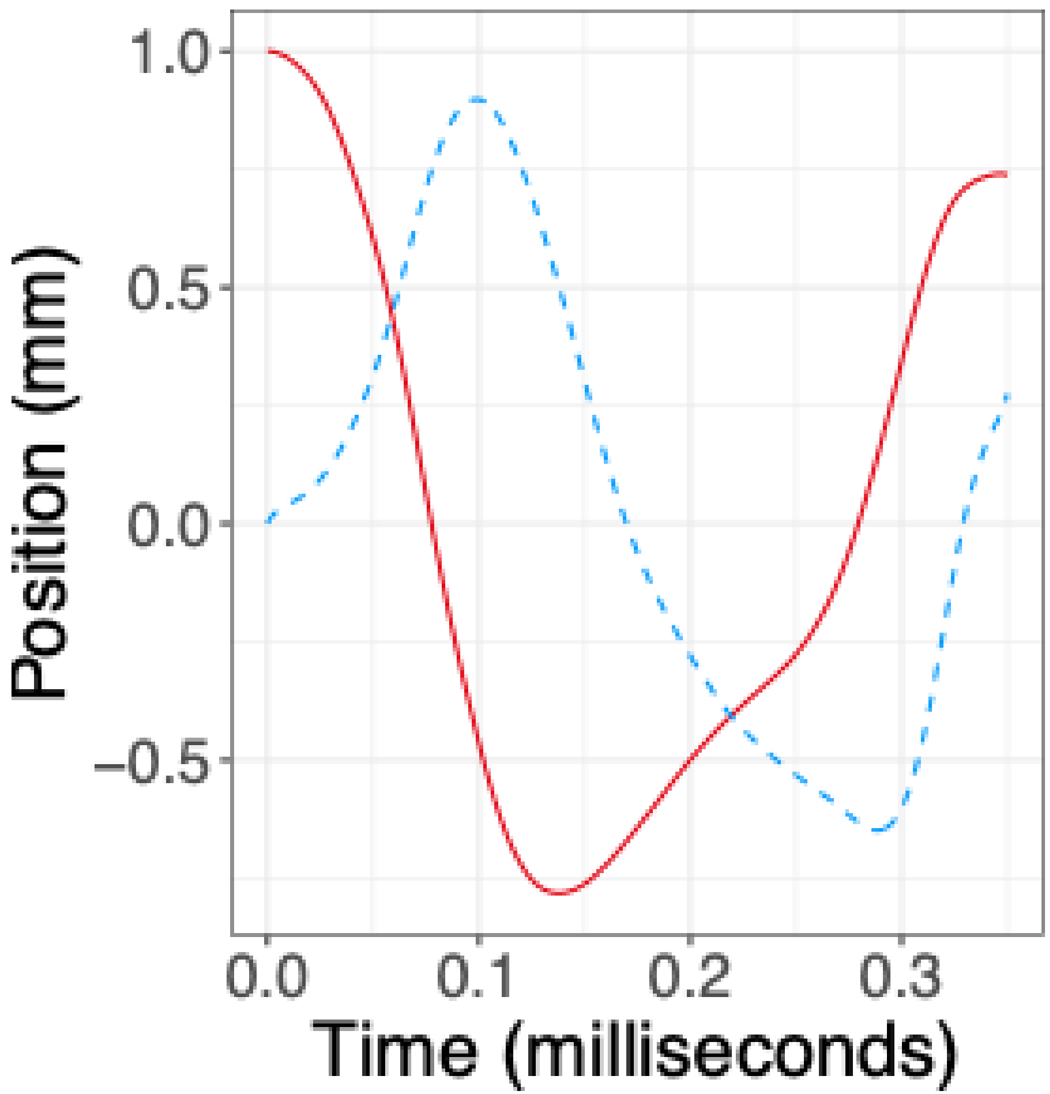
β0
1000
500
0
0.0
0.1
0.2
0.3
Time (milliseconds)



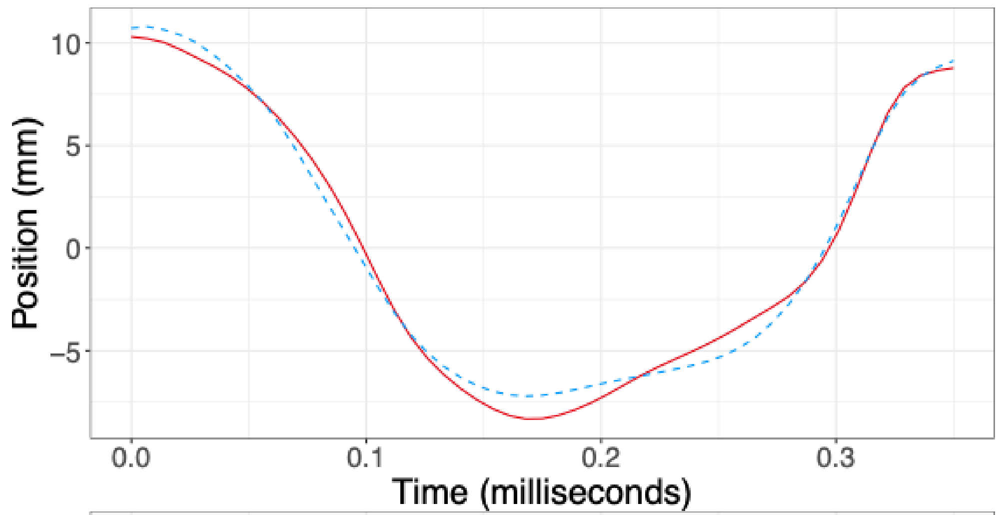
β1
200
0
−200
0.0
0.1
0.2
0.3
Time (milliseconds)



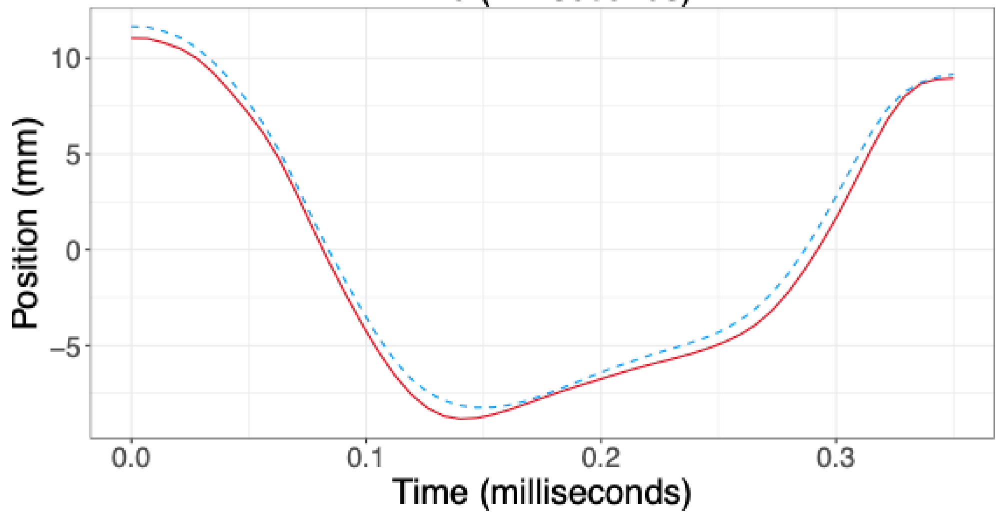
β0
800
600
400
200
0
−200
0.35
0.28
0.14
0.21
0
−200
0
200
β1



1.0
0.5
0.0
−0.5
Position (mm)
0.0
0.1
0.2
0.3
Time (milliseconds)



10
5
0
−5
Position (mm)
0.0
0.1
0.2
0.3
Time (milliseconds)



10
5
0
−5
Position (mm)
0.0
0.1
0.2
0.3
Time (milliseconds)

Lots of stuff to do in ODA

and we need math development for that…

Stop being humble: you’re doing AI!

$\mathbb{R}$ $\mathbb{R}^p$

$\mathbb{N}$ $\mathbb{C}$

$\mathcal{L}^2(\mathbb{R})$

Duck space