Adaptive optimal estimation of irregular mean and covariance functions

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Abstract

Straightforward nonparametric estimators for the mean and the covariance functions of functional data are proposed. The random trajectories are, not necessarily differentiable, have unknown regularity, and are measured with error at discrete design points. The measurement error could be heteroscedastic. The design points could be either randomly drawn or common for all curves. The definition of the nonparametric estimators depends on the local regularity of the stochastic process generating the data. First, a simple estimator of this local regularity which takes strength from the replication and regularization features of functional data is given. Next, the mean and the covariance functions are estimated using the "smoothing first, then estimate" approach. The new estimators achieve optimal rates of convergence. They can be applied with both sparsely or densely sampled curves, are easy to calculate and to update, and perform well in simulations. Simulations built upon a real data example on power consumption illustrate the effectiveness of this approach.

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